Efficient Construction of Behavior Graphs for Uncertain Event Data

Marco Pegoraro, Merih Seran Uysal, Wil M.P. van der Aalst

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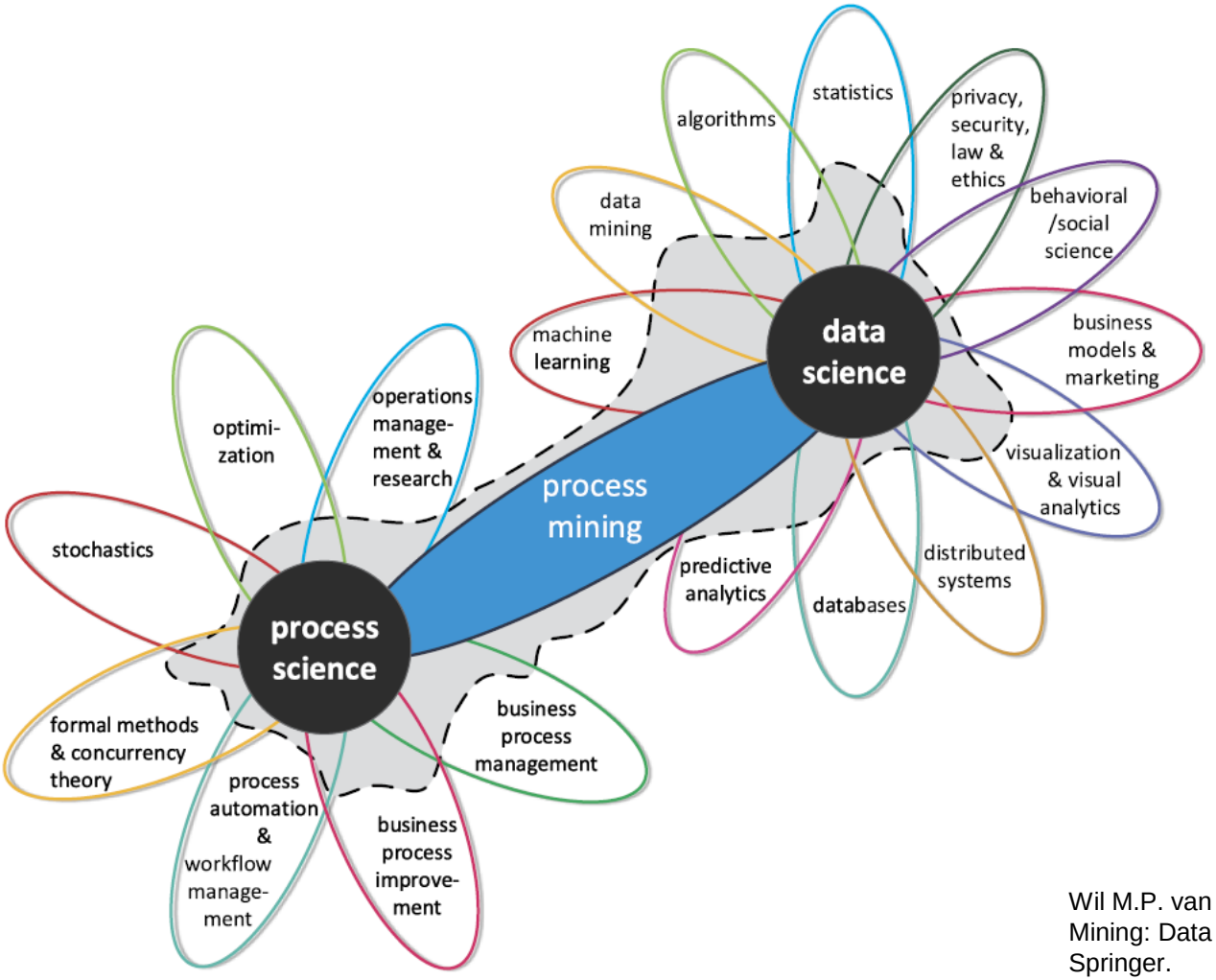
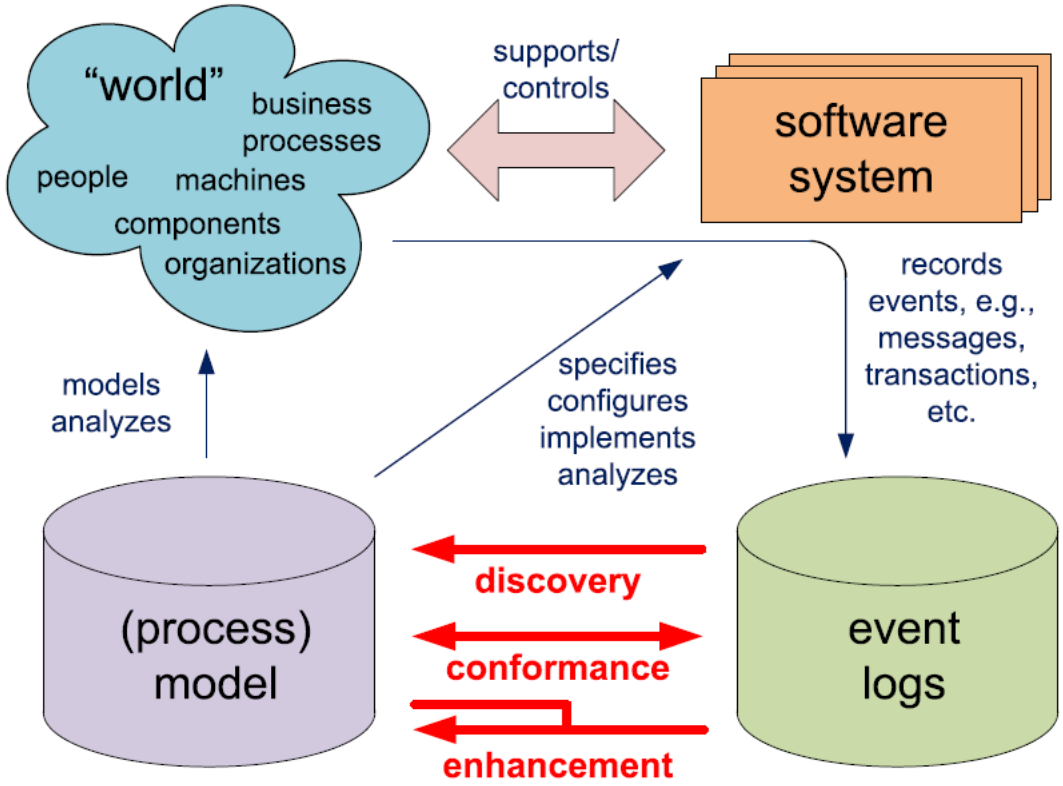
Process Mining



Wil M.P. van der Aalst. “Process Mining: Data science in action." Springer.

Process Mining



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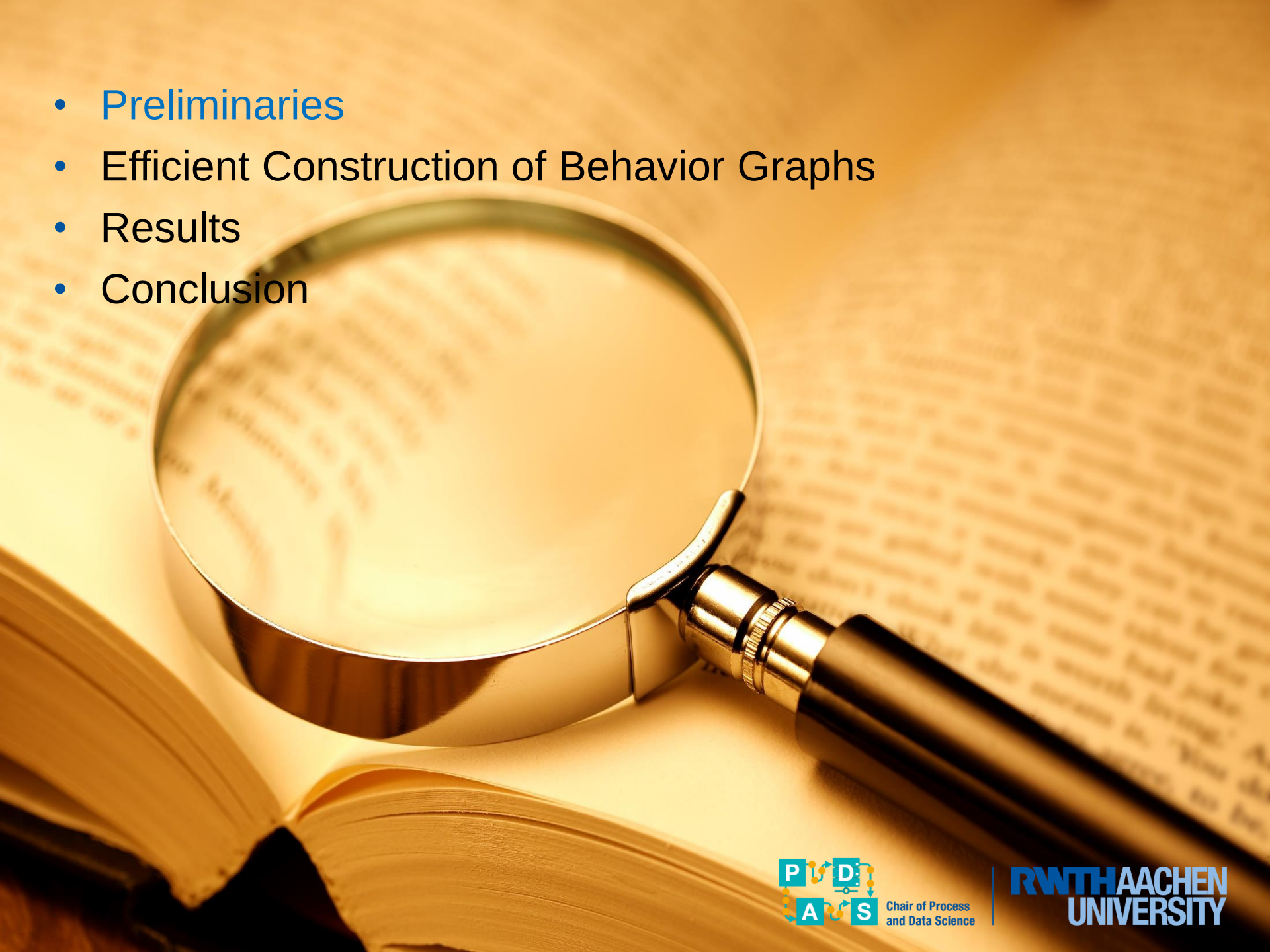

- Preliminaries
- Efficient Construction of Behavior Graphs
- Results
- Conclusion

Event log

Case id	Event id	Properties				
		Timestamp	Activity	Resource	Cost	...
1	35654423	30-12-2010:11.02	register request	Pete	50	...
	35654424	31-12-2010:10.06	examine thoroughly	Sue	400	...
	35654425	05-01-2011:15.12	check ticket	Mike	100	...
	35654426	06-01-2011:11.18	decide	Sara	200	...
	35654427	07-01-2011:14.24	reject request	Pete	200	...
2	35654483	30-12-2010:11.32	register request	Mike	50	...
	35654485	30-12-2010:12.12	check ticket	Mike	100	...
	35654487	30-12-2010:14.16	examine casually	Pete	400	...
	35654488	05-01-2011:11.22	decide	Sara	200	...
	35654489	08-01-2011:12.05	pay compensation	Ellen	200	...
3	35654521	30-12-2010:14.32	register request	Pete	50	...
	35654522	30-12-2010:15.06	examine casually	Mike	400	...
	35654524	30-12-2010:16.34	check ticket	Ellen	100	...
	35654525	06-01-2011:09.18	decide	Sara	200	...
	35654526	06-01-2011:12.18	reinitiate request	Sara	200	...
	35654527	06-01-2011:13.06	examine thoroughly	Sean	400	...
	35654530	08-01-2011:11.43	check ticket	Pete	100	...
	35654531	09-01-2011:09.55	decide	Sara	200	...
	35654533	15-01-2011:10.45	pay compensation	Ellen	200	...

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Traces

Case id	Event id	Properties				
		Timestamp	Activity	Resource	Cost	...
1	35654423	30-12-2010:11.02	register request	Pete	50	...
	35654424	31-12-2010:10.06	examine thoroughly	Sue	400	...
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	35654488	05-01-2011:11.22	decide	Sara	200	...
	35654489	08-01-2011:12.05	pay compensation	Ellen	200	...

A **trace** consists in the set of all the events belonging to a certain case.
It is usually represented by a sequence ordered by timestamp:

1. <register request, examine thoroughly, check ticket, decide, reject request>
2. <register request, check ticket, examine casually, decide, pay compensation>

Example of uncertain trace

An **uncertain trace** is a process trace where some attributes are described with a **range or a set of possible values**.

In this paper, we consider traces with uncertainty on the **timestamp attribute**.

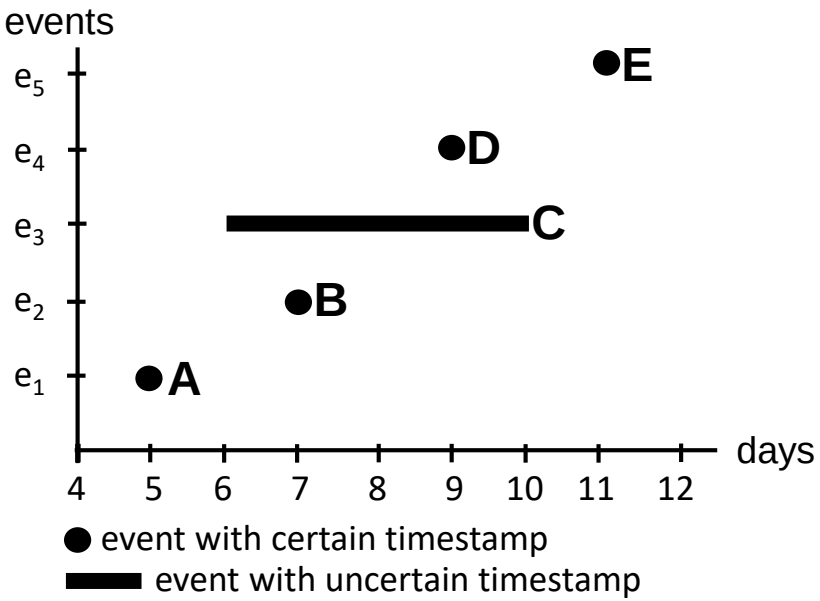
Event ID	Case ID	Timestamp	Activity
e_1	0	05.12.2011	A
e_2	0	07.12.2011	B
e_3	0	**[06.12.2011, 10.12.2011]**	C
e_4	0	09.12.2011	D
e_5	0	11.12.2011	E

The exact timestamp of e_3 belongs to this interval

Marco Pegoraro and Wil M.P. van der Aalst, "Mining Uncertain Event Data in Process Mining," *2019 International Conference on Process Mining (ICPM)*, Aachen, Germany, 2019, pp. 89-96. doi: 10.1109/ICPM.2019.00023.

Realizations of an uncertain trace

Event ID	Case ID	Timestamp	Activity
e_1	0	05.12.2011	**A**
e_2	0	07.12.2011	**B**
e_3	0	[06.12.2011, 10.12.2011]	**C**
e_4	0	09.12.2011	**D**
e_5	0	11.12.2011	**E**



Possible realizations:
<A, B, C, D, E>
<A, B, D, C, E>
<A, C, B, D, E>

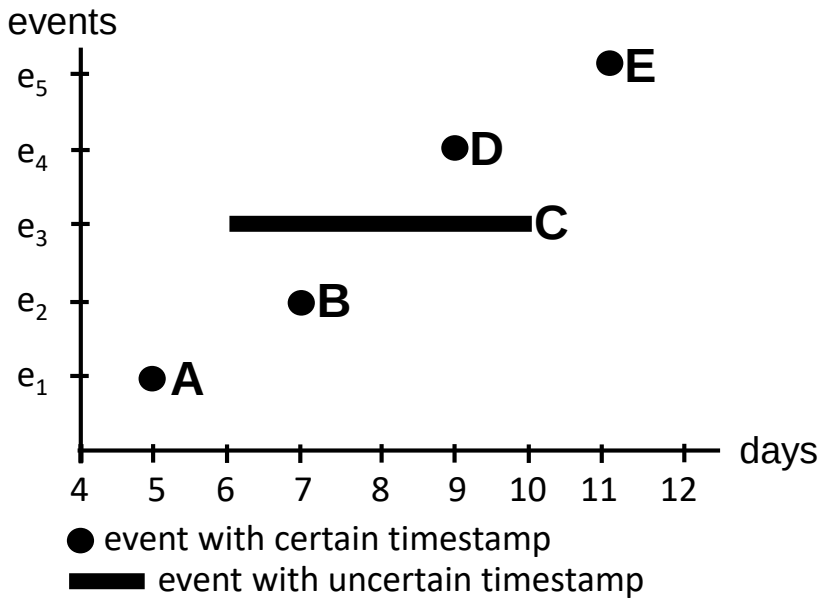
Representation of an uncertain trace

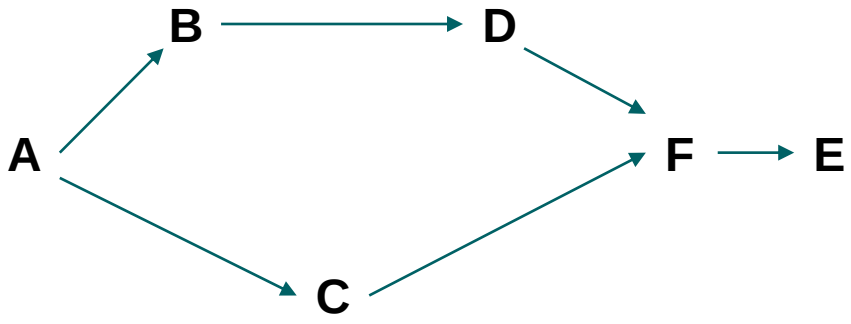
Uncertain traces cannot be represented by a single sequence like regular ones.

Instead, they need to be represented by a **graph** that shows **precedence relationships**.

Behavior graph

Event ID	Case ID	Timestamp	Activity
e_1	0	05.12.2011	**A**
e_2	0	07.12.2011	**B**
e_3	0	[06.12.2011, 10.12.2011]	**C**
e_4	0	09.12.2011	**D**
e_5	0	11.12.2011	**E**





Behavior graph

Obtaining the behavior graph is essential to performing process mining on uncertain event logs.

Behavior graphs allow to query for relationships between activities in the process, a fundamental step of **process discovery** [1].

They are also important to be able to compare the possible behavior of an uncertain trace and a reference model, enabling **conformance checking** over uncertainty [2].

[1] Marco Pegoraro, Merih Seran Uysal, and Wil MP van der Aalst. "Discovering Process Models from Uncertain Event Data." *International Conference on Business Process Management*. Springer, Cham, 2019.
[2] Marco Pegoraro and Wil M.P. van der Aalst, "Mining Uncertain Event Data in Process Mining," *2019 International Conference on Process Mining (ICPM)*, Aachen, Germany, 2019, pp. 89-96. doi: 10.1109/ICPM.2019.00023.

Behavior graph creation

A behavior graph is conceptually simple to obtain:

- Connect event e_1 to event e_2 if they do not overlap and e_1 happened before e_2
- Perform transitive reduction on the resulting graph

Transitive reduction: removing the maximum number of edges from a graph without altering the **reachability** between nodes.

Behavior graph creation

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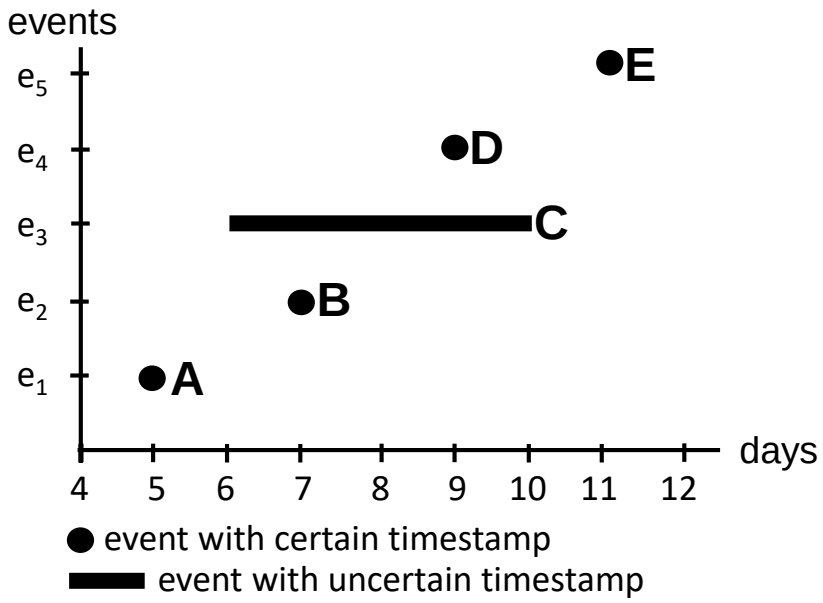

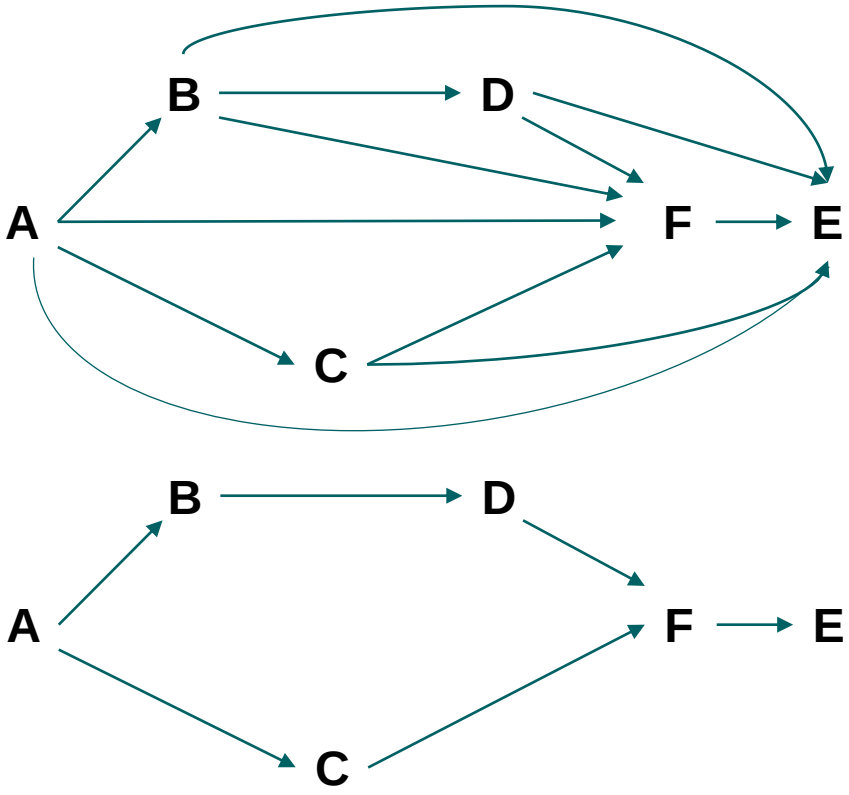

Behavior graph creation

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Transitive reduction: removing the maximum number of edges from a graph without altering the **reachability** between nodes.

Cost for a trace with n events: **$O(n^3)$**

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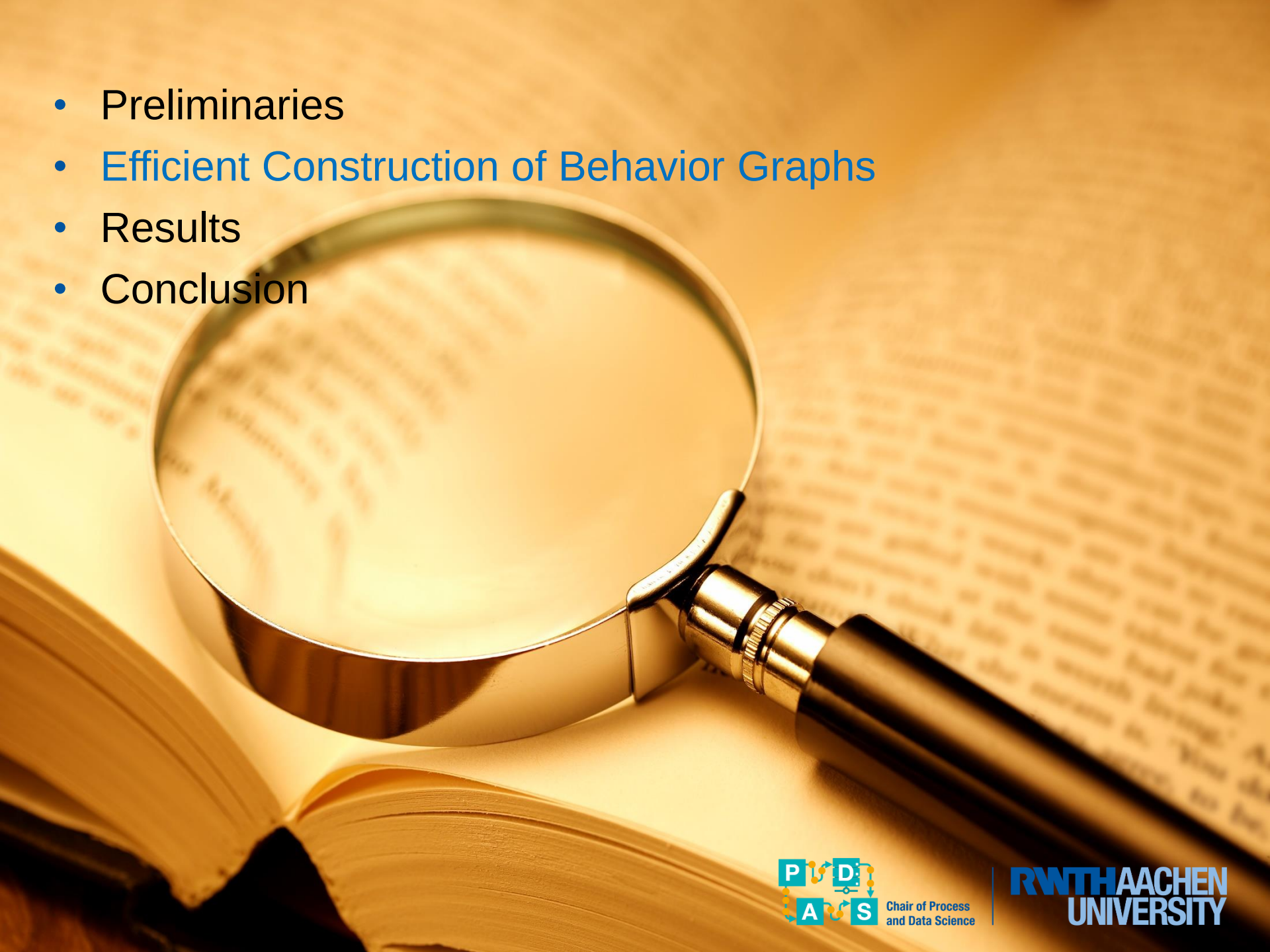

Preprocessing

- Preprocessing step: transform the uncertain trace in a list of activities sorted by timestamp

- In this list, events with an uncertain timestamps become **two entries:** the left and right extreme of the time interval are inserted in the list, labeled as such, with a reference to the original activity label

EventID	Case ID	Timestamp	Activity
e1	0	05.12.2011	A
e2	0	07.12.2011	B
e3	0	[06.12.2011, 10.12.2011]	C
e4	0	[08.12.2011, 11.12.2011]	D
e5	0	09.12.2011	E
e6	0	[12.12.2011, 13.12.2011]	F

Preprocessing

- Preprocessing step: transform the uncertain trace in a list of activities sorted by timestamp

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EventID	Case ID	Timestamp	Activity
e1	0	5	A
e2	0	7	B
e3	0	[6, 10]	C
e4	0	[8, 11]	D
e5	0	9	E
e6	0	[12, 13]	F

Preprocessing

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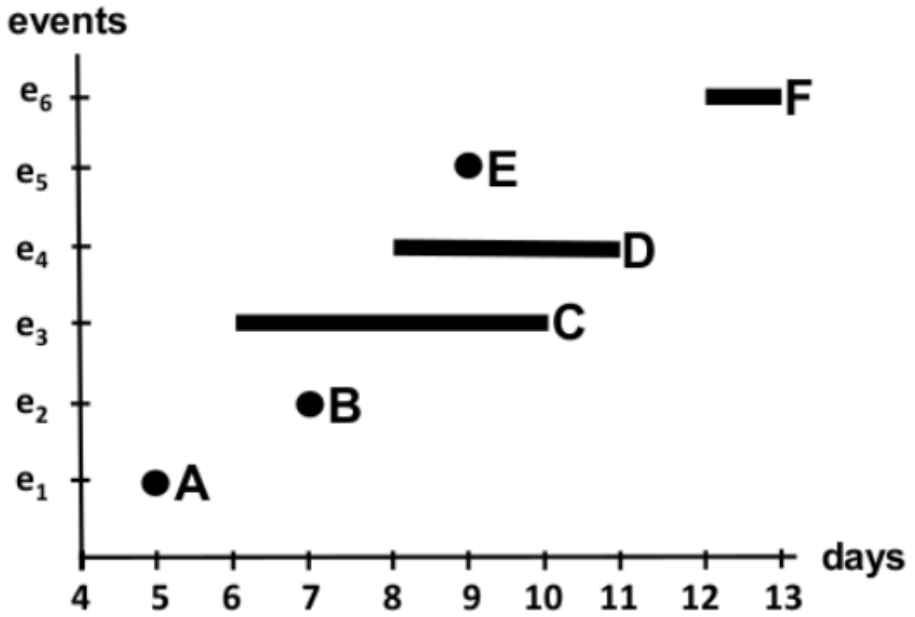
e6	0	[12, 13]	F

A, C_L, B, D_L, E, C_R, D_R, F_L, F_R

Preprocessing

Visiting the list in order is the equivalent of **sweeping** the time diagram in order to «discover» beginning and end of events.



A, C_L, B, D_L, E, C_R, D_R, F_L, F_R

Behavior graph creation algorithm

Input:

list *The preprocessed list of timestamps*

```
for i = 1 to n
   if list[i] is not a left extreme
   for j = i+1 to n
      if list[j] is a left extreme
         connect list[i] node with list[j] node and continue
      if list[j] is a certain timestamp
         connect list[i] node with list[j] node and stop
      if list[j] is a right extreme
         if list[i] node was already connected with list[j] node
            stop
         else
            continue
```

Behavior graph creation: example

$A, C_L, B, D_L, E, C_R, D_R, F_L, F_R$

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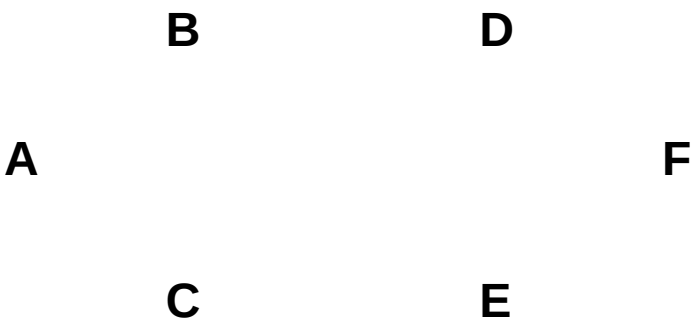

Behavior graph creation: example

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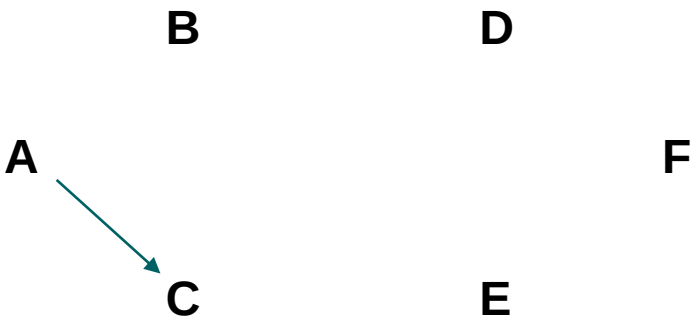

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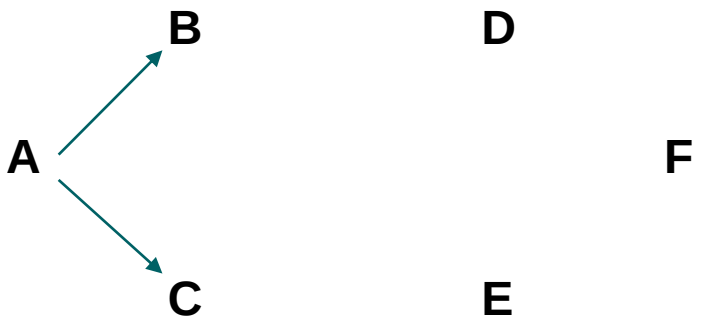


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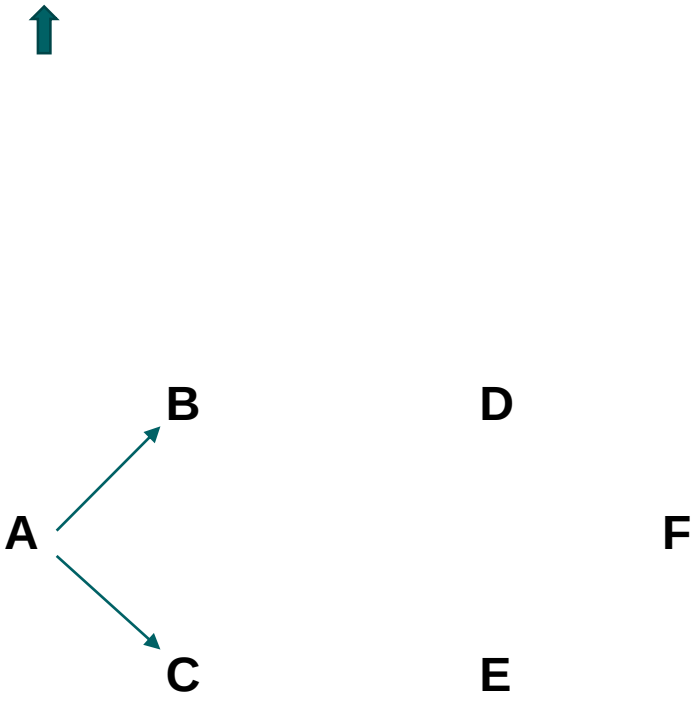
Behavior graph creation: example

$A, C_L, B, D_L, E, C_R, D_R, F_L, F_R$

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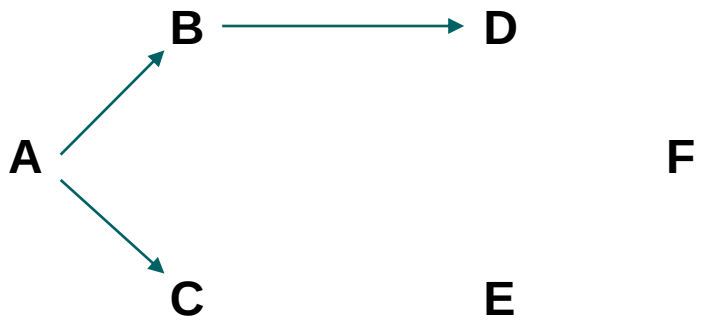


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Behavior graph creation: example

$A, C_L, B, D_L, E, C_R, D_R, F_L, F_R$

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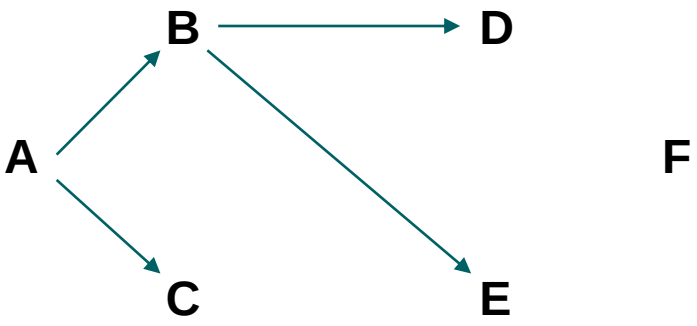

Behavior graph creation: example

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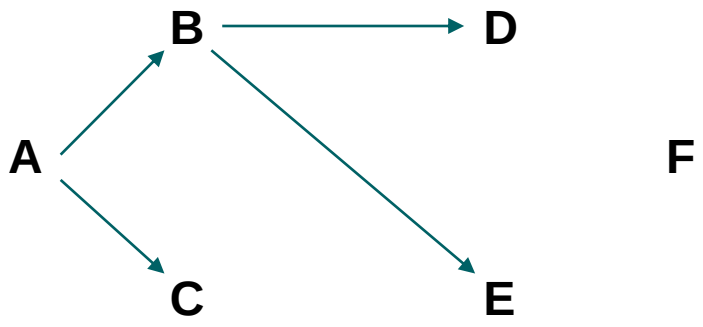


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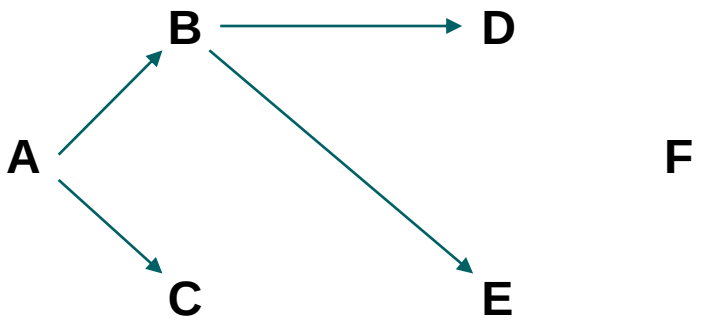


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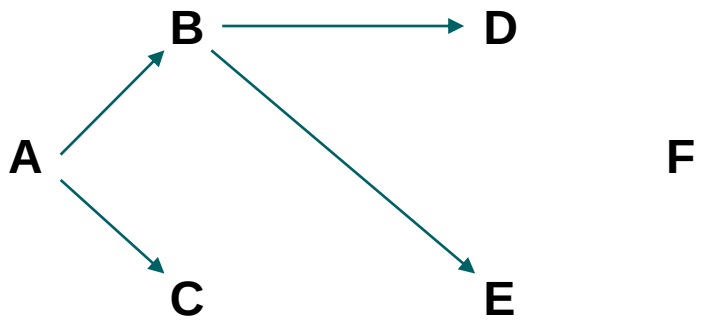


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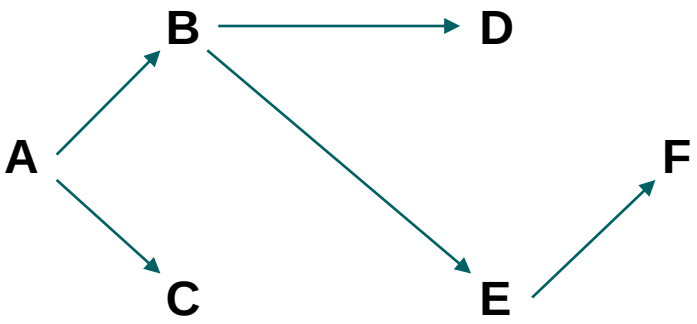

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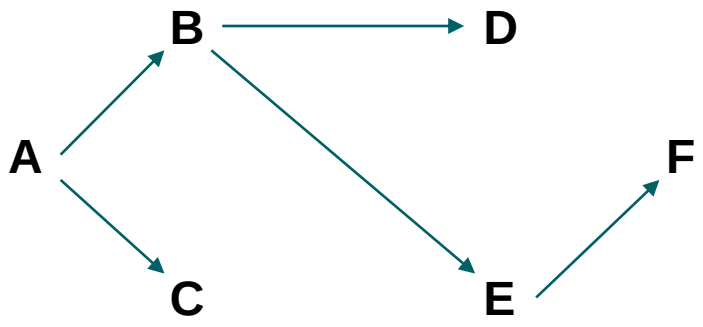


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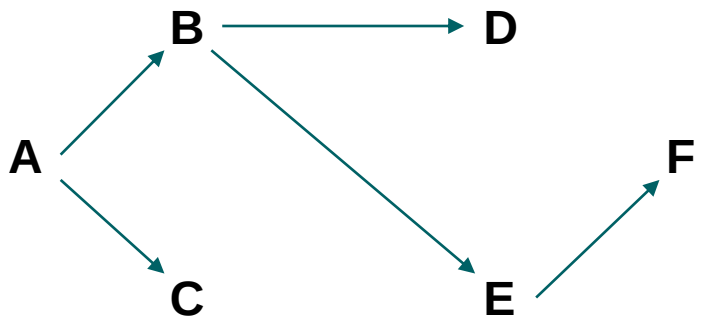


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Behavior graph creation: example

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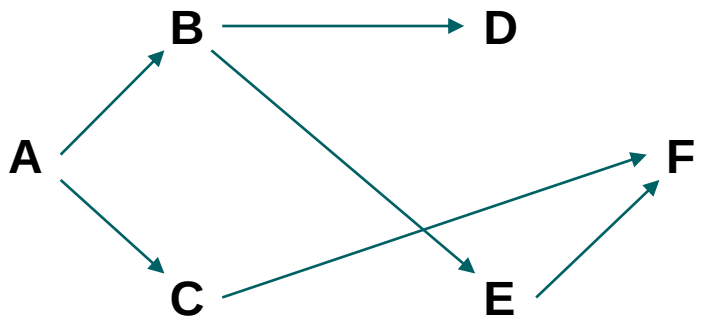


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        if list[j] is a certain timestamp
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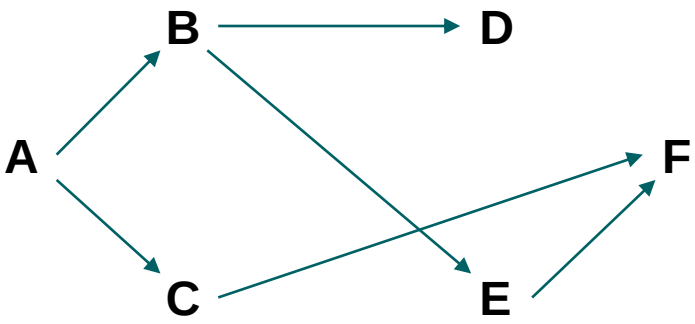
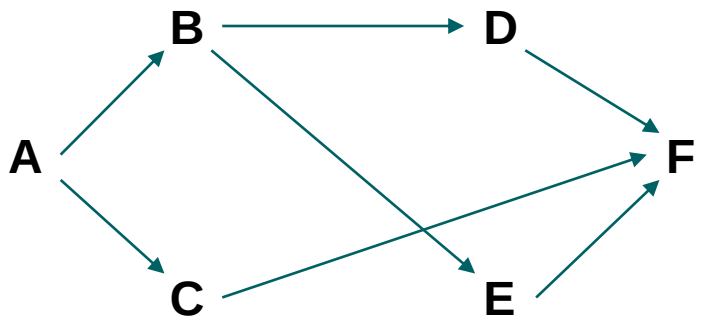
        if list[j] is a right extreme
            if list[i] node was already connected with list[j] node
                stop
            else
                continue
```



Behavior graph creation: example

$A, C_L, B, D_L, E, C_R, D_R, F_L, F_R$

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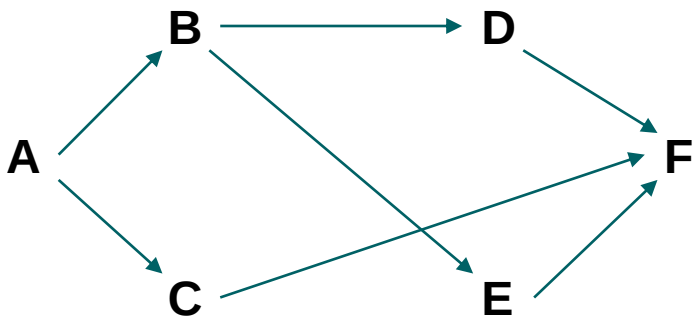


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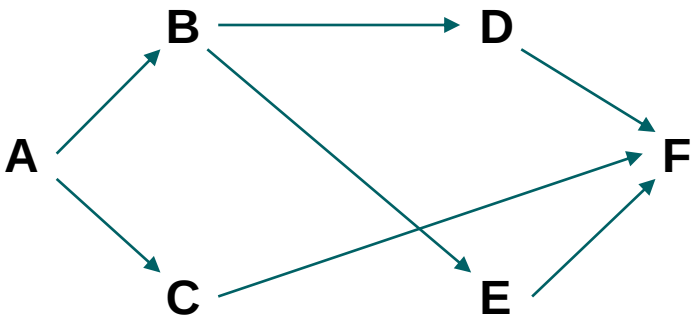
Behavior graph creation: example

$A, C_L, B, D_L, E, C_R, D_R, F_L, F_R$

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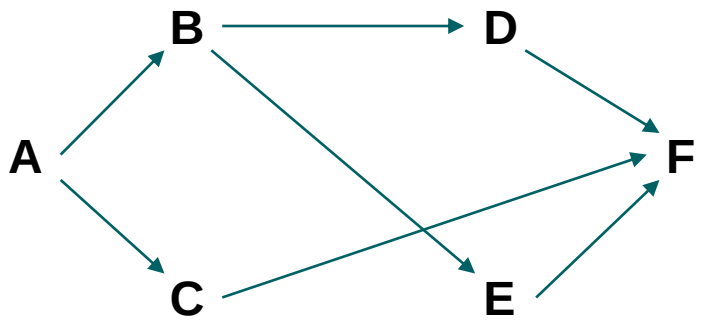
Behavior graph creation: example

$A, C_L, B, D_L, E, C_R, D_R, F_L, F_R$

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                stop
            else
                continue
```
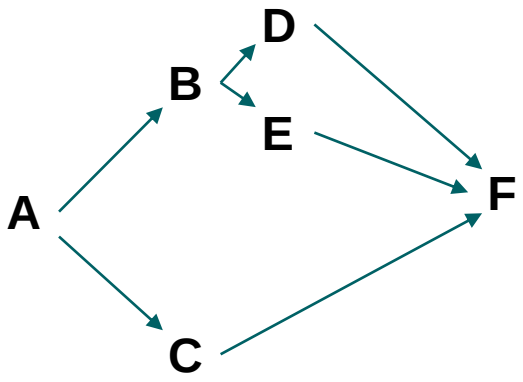

Behavior graph creation: example

EventID	Case ID	Timestamp	Activity
e1	0	5	A
e2	0	7	B
e3	0	[6, 10]	C
e4	0	[8, 11]	D
e5	0	9	E
e6	0	[12, 13]	F



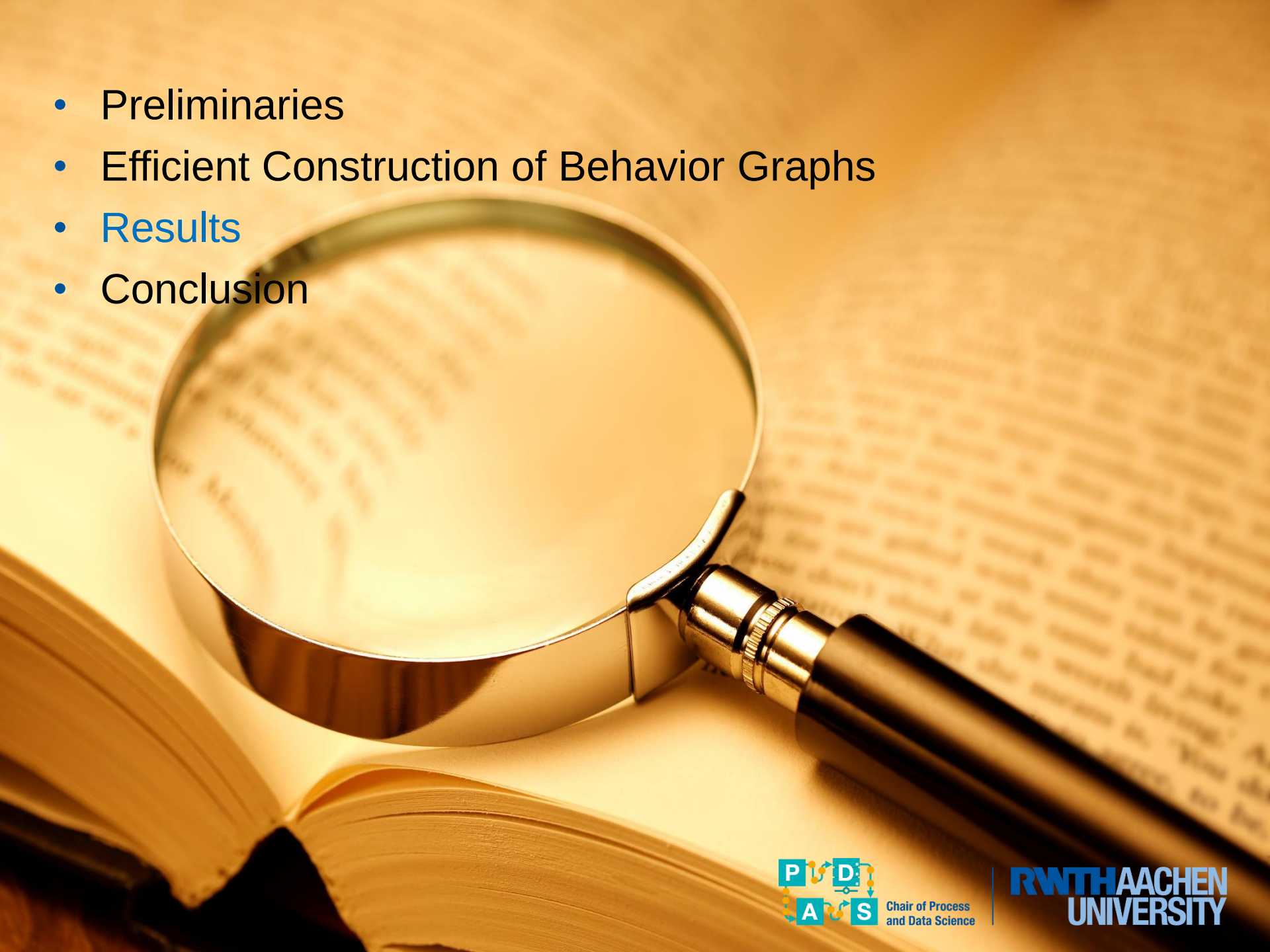
Behavior graph creation: complexity

Let us examine the complexity of this algorithm.

If the uncertain trace has *n* events, the sorted list of timestamp is long at most *2n*.

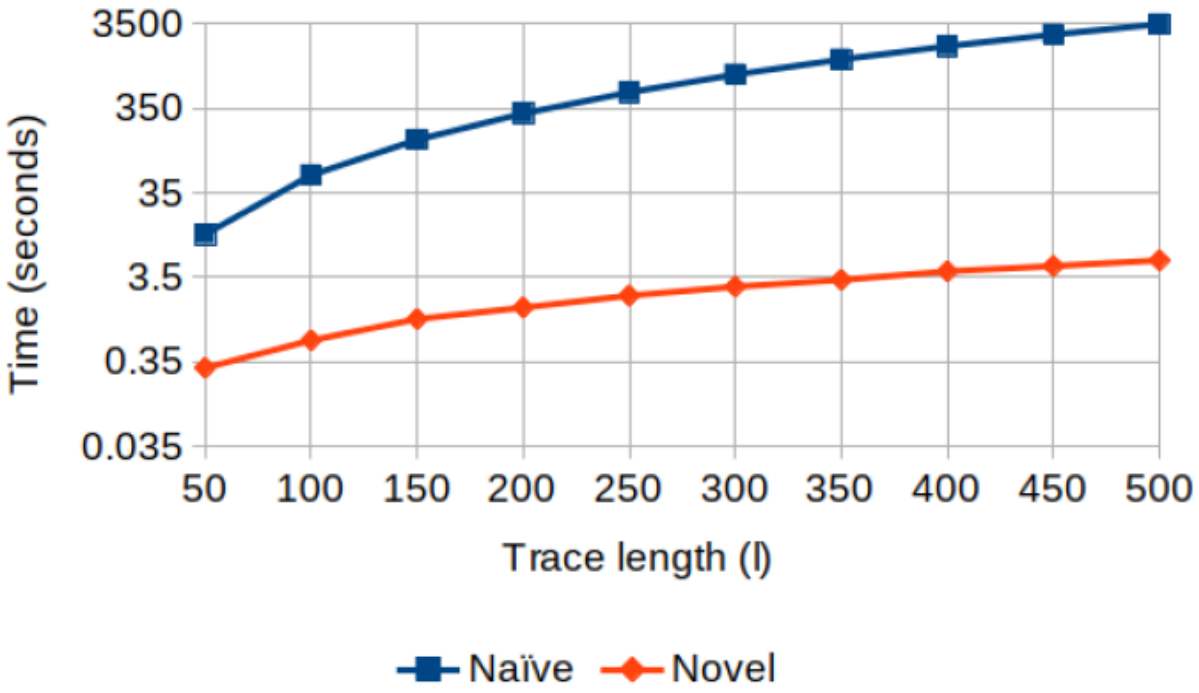
The algorithm sorts this list and then uses two nested loops on it.

The complexity is then $O(2n \cdot \log(2n) + 2n \cdot 2n) =$ **$O(n^2)$.**



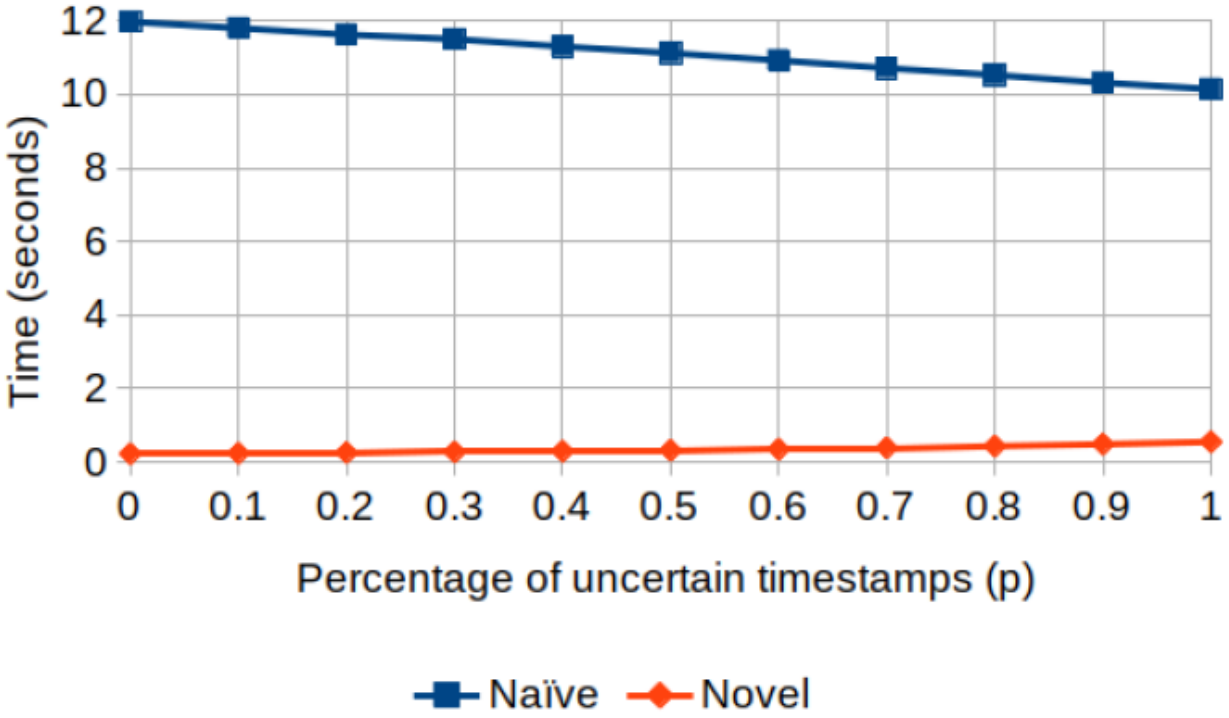
- Preliminaries
- Efficient Construction of Behavior Graphs
- Results
- Conclusion

Behavior graph creation: experiments

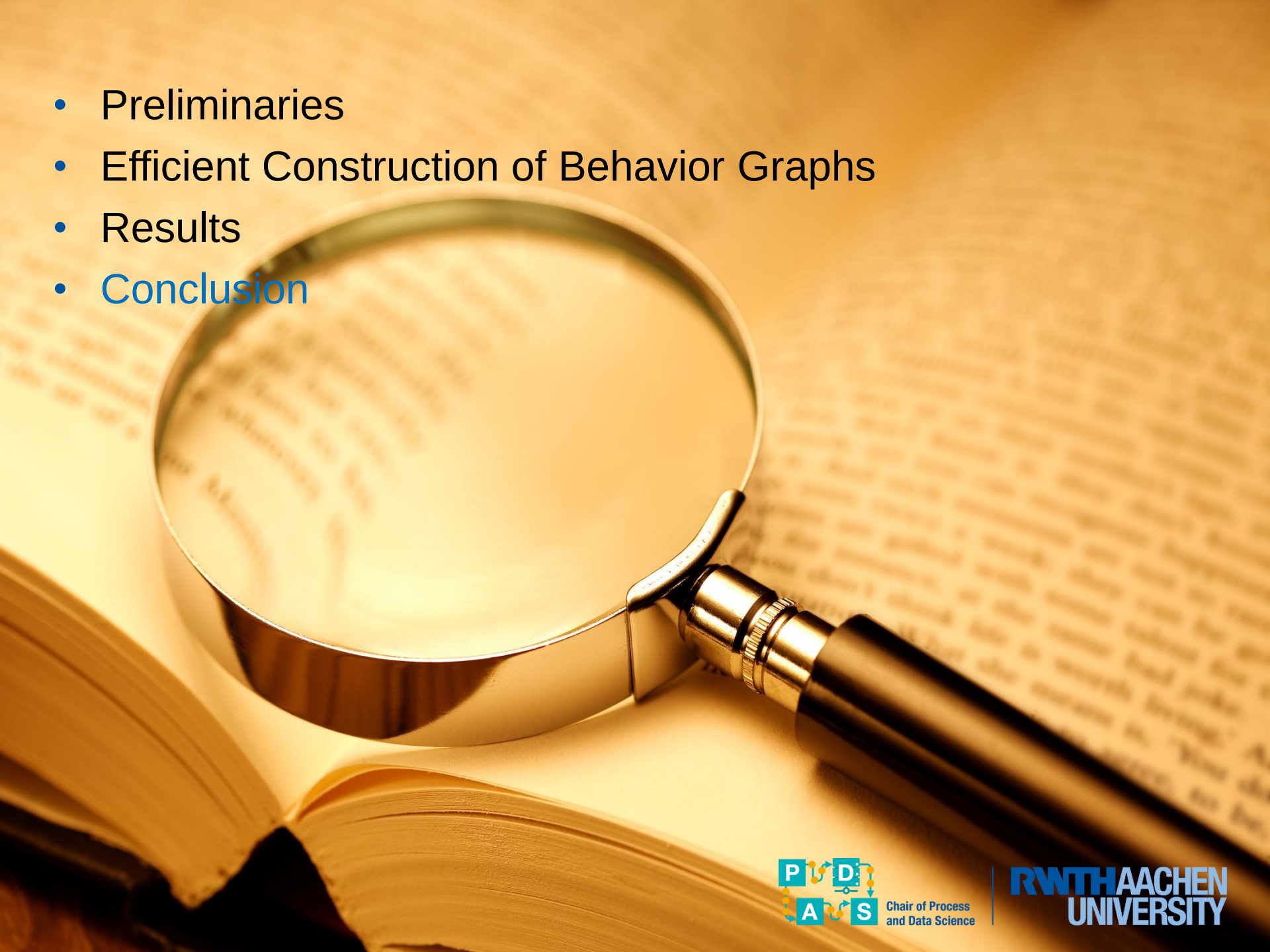


Execution time on an **uncertain log**, in function of the **trace length** (number of traces and number of uncertain events remain constant)

Behavior graph creation: experiments



Execution time on an **uncertain log**, in function of the **percentage of uncertain events** (number of traces and trace length remain constant)



- Preliminaries
- Efficient Construction of Behavior Graphs
- Results
- Conclusion

Conclusion

- The new method of construction for behavior graphs removes a **major bottleneck** in the performance of uncertainty analysis in process mining.

- The running time for conformance and discovery under uncertainty is **strongly reduced** (for typical log dimensions).

- More work needed:
  - Average case analysis
  - More experiments (also in context of discovery and conformance checking)

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Experiments available at
https://github.com/proved-py/proved-core/tree/Efficient_Construction_of_Behavior_Graphs_for_Uncertain_Event_Data

Marco Pegoraro
pegoraro@pads.rwth-aachen.de
www.mpegoraro.net

Thank you!

References

Marco Pegoraro and Wil M.P. van der Aalst, "Mining Uncertain Event Data in Process Mining," *2019 International Conference on Process Mining (ICPM)*, Aachen, Germany, 2019, pp. 89-96. doi: 10.1109/ICPM.2019.00023.